Standard quantum mechanics needs no collapse

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May 21, 2022

1 Introduction

This note (work in progress) has originated in discussion on a Google group
`https://groups.google.com/g/Bell_quantum_foundations`
devoted to Bell's theorem and the interpretation of quantum mechanics. One of the group members, Bryan Sanctuary, insists that two particles leaving a source cannot remain entangled. He claims that the EPR-B correlations cannot remain valid once the particles are separated. At the same time, he claims that the EPR-B correlations do have a local and realistic interpretation once one introduces a hidden, quantum property which he calls hyperhelicity, see the papers linked on his blog posts
`http://blog.mchmultimedia.com/2022/05/09/hyper-helicity-and-the-foundations-of-qm/`,
`http://blog.mchmultimedia.com/2022/04/26/hyperhelicity-the-completion-of-spin/`,
to mention just two of his recent posts on this topic. He believes that in this way all notions of non-local collapse and weirdness can be banished from quantum mechanics. Another participant, Alexey Nikulov, also thinks that conventional quantum mechanics is wrong, and disbelieves in non-locality and collapse,
`https://www.researchgate.net/publication/326200568_Challenging_local_realism_and_the_crisis_of_physics`.

My first purpose in this note is to make it clear that "collapse of the wave-function" is an interpretational optional extra, not needed in order to apply quantum mechanics in practice. No interpretation of what is going on behind the scenes is needed if one's purpose is to *describe* nature. Interpretations of quantum mechanics generally add some kind of story to the basic mathematical rules, in an attempt to *understand* nature.

My second purpose is to write out the mathematical content of Sanctuary's claims concerning hyper-helicity. The second of his above mentioned blog posts gives the most details on his model. The first expounds on what he sees are the consequences. Sanctuary feels that his interpretation of his mathematical formalism will revolutionise understanding of quantum entanglement. Clearly he has a long way to go, but I hope my own struggle to understand what he is doing will be helpful.

2 Notation and definitions

First I recall some standard definitions and fix my notation.

I will use "cdot" ($\cdot$) to denote inner product, applied to 3-vectors, but also to a 3-vector and an ordered list of three operators. I will use "otimes" ($\otimes$) to denote outer product. An identity matrix will be denoted by I, its dimension will be evident from the context.

I will use the notation $(|1\rangle, |2\rangle)$ to stand for the standard orthonormal basis of $\mathbb{C}^2$. Following on from this, $|1,2\rangle := |1\rangle \otimes |2\rangle$ stands for an element of $\mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$, etc. The singlet state vector is the vector $|\Psi\rangle = \{|1,2\rangle - |2,1\rangle\}/\sqrt{2}$. The corresponding density matrix is $\rho = |\Psi\rangle\langle\Psi| = \{|1,2\rangle - |2,1\rangle\}\{\langle 1,2| - \langle 2,1|\}/2$. It is a 4×4 complex matrix: non-negative, trace 1, self-adjoint (Hermitean). It moreover has rank 1. It is a rank-one projector.

The measurement directions are 3-vectors of length one, $\boldsymbol{a}$ and $\boldsymbol{b}$. The Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Define $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Define $\sigma_{\boldsymbol{a}} = \boldsymbol{a} \cdot \boldsymbol{\sigma}$ and $\sigma_{\boldsymbol{b}} = \boldsymbol{b} \cdot \boldsymbol{\sigma}$; these are 2×2 complex matrices.

Consider a two-particle system in the singlet state, on which we measure the spins in directions $\boldsymbol{a}$ and $\boldsymbol{b}$. The two observables being measured on the composite system are $\sigma_{\boldsymbol{a}} \otimes I$ and $I \otimes \sigma_{\boldsymbol{b}}$. They commute, and their product is $\sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}$. Following the rules of conventional quantum mechanics, the statistics of measuring each spin separately and multiplying the two outcomes is the same as the statistics of measuring the product observable $\sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}$. The expectation value of this product is therefore $\mathrm{trace}(\rho\, \sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}) = \langle\Psi|\sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}|\Psi\rangle$. I call this function of $\boldsymbol{a}$ and $\boldsymbol{b}$ "the singlet correlation".

Quantum mechanics does tell us more. It is easy to check that $\sigma_{\boldsymbol{a}}^2 = I$, $\sigma_{\boldsymbol{b}}^2 = I$. The eigenvalues of $\sigma_{\boldsymbol{a}} \otimes I$ and of $I \otimes \sigma_{\boldsymbol{b}}$ must therefore be ± 1 since the squares of the (real) eigenvalues all equal $+1$. One can compute the two mean values $\mathrm{trace}(\rho\, \sigma_{\boldsymbol{a}} \otimes I) = 0$ and $\mathrm{trace}(\rho\, I \otimes \sigma_{\boldsymbol{b}}) = 0$. Knowing the fact that the joint measurement of the two observables takes values in the set of four possible joint outcomes $\{(\pm 1, \pm 1)\}$, that the mean values are zero, and knowing the mean of the product, we can easily compute the complete probability distribution. The probability of outcome (x, y) is $(1 - xy\, \boldsymbol{a} \cdot \boldsymbol{b})/4$ for $x, y = \pm 1$.

We assumed the trace rule for computation of mean values of observables. Notice that a collection of commuting observables can be expressed as a function of one single observable. The probability distribution of the measured values of a function of an observable is equal to the probability distribution of the same function of the measured values of the observable. Measurement of an observable results in observation of an eigenvalue of the observable.

We could alternatively have started by assuming a generalization of the Born rule to the situation of the joint measurement of a collection of commuting observables. We could then have easily derived the other just mentioned properties. The important thing to note is that these rules are all that are needed to derive the usual results on quantum teleportation, the GHZ experiment, and so on. First main point of this paper:

The notion that there is a wave function which collapses on measurement was nowhere used in these computations

One also has rules for the state of a system after an ideal measurement of an observable, and hence for the joint probability distribution of the outcomes of a sequence of measurements. One might consider that these rules do involve a "collapse" assumption. However, this is an illusion. Certainly, after a measurement has been made and its outcomes are available to some agent, that agent's predictions about results of future measurements will be different from what they would be, without the knowledge of the intermediate outcomes. The rules together just allow one to compute the joint probability distribution of the results of a sequence of ideal measurements, by decomposing it in Markov fashion. There is no implication that the physical system under study has changed in a non-local way, though the rules certainly do bring that suggestion uncomfortably to mind.

The usual colourful language involving non-local collapse of the wave function can be thought just to be a description of a useful computational tool, not a description of physical changes to something existing in physical reality. The only thing assumed to exist are measurement outcomes, and the theory allows us to compute probability distributions of their outcomes, also in complex, composite, sequential, experimental set-ups. One can compute what one needs to know by *pretending* that the wave function collapses as suggested by the von Neumann-Lüders extension of the Born law are somehow real. One gets the right answer, as directly as possible. There is however no need to think of wave function collapse as being something physical (and necessarily non-local). Such thinking is an optional extra. Some people find it distasteful. Tastes differ. I think it can be usefully be thought of as one of those *lies for children* which needs to be seen in a different light as one gains maturity and knowledge.

3 Helicity

Bryan Sanctuary has noticed that we can expand

$$\rho = \{|1,2\rangle\langle 1,2| + |2,1\rangle\langle 2,1|\}/2 \;-\; \{|1,2\rangle\langle 2,1| + |2,1\rangle\langle 1,2|\}/2$$

$$=: \;\rho_{\text{collapsed}} - \tau_{\text{remainder}}.$$

Notice the minus sign and the fact that the second term is not a density matrix. But the first term is. It is the state of a system which with probability half has the first spin up and the second spin down, and with probability half the other way round. It is a mixed state, being a mixture of two rank one projectors.

This allows Bryan Sanctuary to write the singlet correlations as the difference of two terms

$$\langle\Psi|\sigma_a \otimes \sigma_b|\Psi\rangle \;=\; \text{trace}(\rho_{\text{collapsed}}\,\sigma_a \otimes \sigma_b) - \text{trace}(\tau_{\text{remainder}}\,\sigma_a \otimes \sigma_b),$$

where the first term is the correlation which would be observed if the two particles' joint state had collapsed on separation. He attempts to give the remainder term a physical interpretation by introducing "anti-Hermitean observables". As operators, such objects have purely imaginary eigenvalues. Sanctuary considers them as quantum hidden variables.

My first guess was that his approach would be to write the minus sign as the square of the square root of minus one, and to multiply both of the two occurrences of the vector of observables $\boldsymbol{\sigma}$ in the "trace rule formula" trace($\tau_{\text{remainder}}\,\sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}$) by i, taking as it were the real 3-vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ to the "outside" of the whole expression, two occurrences of $i\boldsymbol{\sigma}$ to the inside. This can be neatly expressed in higher-order tensor notation, and appears to be related to calculations in quantum field theory. He calls $i\boldsymbol{\sigma}$ the helicity. He calls all observables and anti-observables "elements of reality" and he asserts that this model is "local" and "realistic". Having enlarged the meanings of the words in the dictionary of quantum mechanics, he can now assert that helicity accounts for the singlet correlations and that his model is local and realistic.

He also says he has disproved Bell's theorem through a counter-example but so far he has not provided any example. Bell's theorem states that a classical local hidden variables theory cannot reproduce certain quantum correlations without violating locality (or worse – superdeterminism). It does not say anything about what Sanctuary asks us to call a quantum local hidden variables theory. At present, we have an introduction of new terminology which allows him to state that the previously unrecognised quantum hidden variable helicity accounts for the violation of Bell's inequalities in the EPR-B situation. It does not reproduce existing basic quantum mechanical predictions. Whether or not it can explain existing experimental results is unclear.

I plan to rewrite Sanctuary's analysis in notation which I can understand. I have no doubt that Sanctuary's computations are correct, in the sense that he correctly derives the usual formula for the singlet correlations from a more complicated quantum description. However, he seems to be saying that once the particles have separated the state is no longer entangled and therefore that the correlations will change. However, in that case, there would exist a LHV description of the experiment, and the Bell iequalities would not be violated. He doesn't explain how they are violated in the 2015 experiments.

Here is a first step. Writing just τ for the anti-Hermitean operator $\tau_{\text{remainder}} = \{|1,2\rangle\langle 2,1| + |2,1\rangle\langle 1,2|\}/2$, one obtains that the remainder term in the correlation is

$$-\text{trace}(\tau\,\sigma_{\boldsymbol{a}} \otimes \sigma_{\boldsymbol{b}}) \;=\; \boldsymbol{a} \cdot \Bigl(\text{trace}\bigl(\tau\,(i\boldsymbol{\sigma}) \otimes (i\boldsymbol{\sigma})\bigr)\Bigr) \cdot \boldsymbol{b}.$$

The 4×4 matrix τ has two indices, and inside the largest round brackets we are here actually computing a 3×3 matrix of the trace of the products of outer products of pairs of spin matrices with τ. So far, Bryan has declined to read this paper and to tell me whether or not I am on the right track.

To be done: Check this against Sanctuary's definitions and derivation.